Non-linear dynamic analysis of RC slender structures subjected to wind loading based on experimental data

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ABSTRACT

The goal of this paper is to propose a non-linear dynamic model based on experimental data and NBR-6123 (ABNT, 1988) to accomplish a non-linear dynamic analysis of slender structures subjected to wind loading. Several tests were conducted do assess the effective stiffness of slender structures as function of the internal loads. Once one equation to represent the real stiffness was obtained, the dynamic analysis was conducted. At first was computed the static response given by the mean wind speed. In this part of the problem was considered the concept of effective stiffness to represent the physical non-linearity of material and a P-Delta method to represent the geometrical non-linearity. Considering the final stiffness obtained in that P-Delta method was computed the dynamic response given by the floating wind speed, according to the discrete dynamic model given by NBR-6123. A 40 m RC telecommunication tower was analyzed and the results obtained were compared with those given by linear static and dynamic models. The conclusion is that the non-linear dynamic analysis proposed here leads to values of internal loads 15% larger than the traditional linear dynamic analysis and 50% larger than the static analysis.

Keywords: Telecommunication tower, non-linear dynamic analysis, wind loading, reinforced concrete, optimization, experimental data, effective bending stiffness.

1 INTRODUCTION

The models proposed by Brazilian Code NBR-6123-87 (ABNT, 1987) to accomplish a dynamic analysis of structures subjected wind loading are based on linear dynamic models. In RC structures where the effective stiffness changes continuously due to non-linear material behavior and the level of strength, linear models could not describe precisely the structure behavior. Computation of cross-sections properties, and consequently the displacements and internal loads, in slender RC structures subjected to wind loading is a very difficult task because as the loads change along time, cross-sections properties change too. Which stiffness considers? Wind speed is defined by two components, one is the mean wind speed and another is the floating wind speed. Mean wind speed applies on the structures static loads, while floating wind speed applies dynamic loading. The models given by NBR-6123-87 (ABNT, 1987) are based on linear dynamic models, in other words, they consider a constant stiffness along time, what does not happen in practice.

In this work the authors analyze a pre-fabricated 40 m RC telecommunication tower (Fig. 1) similar to others erected at Minas Gerais and Espírito Santo states of Brazil. For the effects of the mean wind speed on structure, the authors consider a non-linear behavior. In this phase, a P-Delta effect will be considered on the structure. In each iteration, the effective stiffness is given by Brasil and Silva (2004). After this method converging, we initiate the computation of the dynamic effects of wind given by the floating wind speed. The authors consider that the structure
vibrates around an equilibrium position. This position is that one gave by the last iteration of P-Delta method. Then, the natural modes and frequencies of vibration are computed considering the effective stiffness given by the last iteration of P-Delta method. Once the natural shapes and frequencies are known, the dynamic analysis can be done according to NBR-6123-87 (ABNT, 1987). The sum of the static, given by P-Delta method with Brasil and Silva (2004) curves, and dynamic components, provided by the discrete dynamic model of NBR-6123-87 (ABNT, 1987), gives the structure behavior.

Figure 1. Typical RC telecommunication tower: original and discretized structure

2 NON-LINEAR DYNAMIC ANALYSIS

2.1 Linear Static Analysis (LSA)

According to NBR-6123-87 (ABNT, 1987) \( V_0 \) (meters per second) is the mean wind speed computed on 3 seconds, at 10 meters above ground, at a plain terrain with no roughness, with recurrence of 50 years. The topographic factor is \( S_1 \), while the terrain roughness is given by factor \( S_2 \), which is a function given by

\[
S_2 = bFr(z/10)^p
\]  

(1)

where \( b, p \) and \( Fr \) are factors which depend on the terrain characteristics, and \( z \) is the height above ground in meters. The statistic factor is \( S_3 \). Both \( S_1, S_2 \) e \( S_3 \) are given by tables in Brazilian Code NBR-6123-87 (ABNT, 1987). The characteristic wind speed (meters per second) and the wind pressure (Pascal) are respectively

\[
V_k = V_0 \cdot S_1 \cdot S_2 \cdot S_3 \quad \text{and} \quad q = 0.613 \cdot V_k^2.
\]  

(2)

The wind load (Newton) on an area \( A \) (projection on a vertical plane of a given object area in square meters) is computed as

\[
F = q \cdot C_a \cdot A,
\]  

(3)

where \( C_a \) is the aerodynamical coefficient. The Brazilian Code NBR-6123-87 (ABNT, 1987) presents tables for \( C_a \) values.
2.2 Linear Dynamic Analysis (LDA)

According to NBR-6123-87 (ABNT, 1987), for the j-th degree of freedom, the total load $X_j$ due to direct along wind is the sum of the mean and floating load given by:

$$X_j = \bar{X}_j + \dot{X}_j$$

where the mean load $\bar{X}_j$ is:

$$\bar{X}_j = \bar{q}_o b^2 c_j A_j \left( \frac{z_j}{z_r} \right)^2 p,$$

being

$$\bar{q}_o = 0.613 P_r^2$$

and

$$P_r = 0.69 V_o S_5$$

($q_o$ in $N/m^2$ and $V_r$ in $m/s$),

where $b$ and $p$ indicated in Table 20 of NBR-6123-87 (ABNT, 1987); $z_r$ is the level of reference, equal to 10 meters in this work; $V_r$ is design wind speed corresponding to the mean speed during 10 minutes at 10 meters above the ground level, for a terrain roughness ($S_2$) category II.

The floating component $\dot{X}_j$, is given by:

$$\dot{X}_j = F_i \psi_j \phi_j$$

where

$$\psi_j = \frac{m_i}{m_0}, \quad F_i = \bar{q}_o b^2 A_i \sum_{i=1}^n \beta_i \phi_i \xi$$

and

$$\beta_i = C_{ai} A_i \left( \frac{z_i}{z_r} \right)^p$$

being $m_i$, $m_0$, $A_i$, $A_0$, $\xi$ and $C_{ai}$, respectively, the lumped mass at the i-th degree of freedom, a reference mass, the equivalent area at the the i-th degree of freedom, a reference area, the dynamic amplification coefficient (Fig. 17 of NBR-6123-87 (ABNT, 1987)) and the area $A_i$ aerodynamical coefficient.

Note that $\phi = [\phi_i]$ is a given mode of vibration. To compute $\phi_i$ and $\xi$ is necessary to consider the structure mass and stiffness. The lumped mass can be easily calculated by summing the mass around an influence region of the node. The total homogenized moment of inertia of the cross-section is given by

$$I_{total} = I_s + I_{s, hom}, \quad \text{where} \quad I_{s, hom} = I_s \left( \frac{E_s}{E_{s, sec}} - 1 \right) \quad \text{and} \quad E_{s, sec} = 0.9 \times 6600 \sqrt{f_{ck}} + 3.5 \quad (MPa),$$

being $E_s$, $E_{s, sec}$, $I_s$, $I_{s, hom}$, $I_i$ and $f_{ck}$, respectively, the elasticity modulus of steel, the secant elasticity modulus of concrete (NBR-6118-78 (ABNT, 1978)), moment of inertia related to the structure axis of the total longitudinal steel area, the homogenized moment of inertia of the longitudinal steel area, the moment of inertia of the total cross-section area and the characteristic compressive resistance in $MPa$ at 28 days old concrete. Since this model is based on linear dynamic models, we consider the cross-section moment of inertia the total stiffness, such as:

$$I = I_{total},$$

of each section to compute stiffness matrix of the structure. This assumption may be justified because if this is a linear elastic model, any cross-section damage can be considered in this analysis, so the stiffness to be considered must be the total stiffness.
When $r$ modes are considered in the analysis, the combination of these modes, for a given dynamic variable $\hat{Q}$, is computed as

$$\hat{Q} = \left[ \sum_{k=1}^{r} \hat{Q}_k \right]^{1/2} \quad \text{and} \quad Y_i = \frac{1}{3} X_i$$  \hspace{1cm} (11)

is transversal dynamic load.

### 2.3 Non-Linear Dynamic Analysis (NDA)

As stated before, the loads due to the wind speed present two components, the static loads due to mean wind speed and the dynamic loads due to the floating wind speed. The static loads are computed as given in Eq. (5) and (6). We call the first results obtained using these equations as the first order static internal loads. At this point, we consider that the structure under those static loads is subjected to the P-Delta Effect. The static displacements ($\vec{\delta}_{i,j}$), at the $i$-th node and the $j$-th iteration of the P-Delta method, are computed considering the effective stiffness. Differently of what occurs in Section 2.2, we consider the following expressions to compute the moment of inertia (Brasil and Silva, 2004) at the $i$-th node and the $j$-th iteration of the P-Delta method:

$$I_{i,j} = I_{EF,i,j} = w_{i,j} I_{total,i} \quad \text{where} \quad w_{i,j} = w(x_{i,j}) \quad \text{and} \quad x_{i,j} = \frac{M_{k(i,j)}}{M_u},$$  \hspace{1cm} (12)

being $I_{EF}, w, x, M_k$ and $M_u$, respectively, the effective moment of inertia, the parameter of effective stiffness, the level of strength, the working bending moment due to mean wind speed and the ultimate code based moment of a given cross-section. In Eq. (12) we consider that the damage occurred in the cross-sections is represented by the effective stiffness concept.

Finally, the P-Delta effect is computed, at the $i$-th node and the $j$-th iteration of the P-Delta method, as

$$\Delta M_{k(i,j)} = \Delta N_{k(i,j)} (\vec{\delta}_{i,j} - \vec{\delta}_{i(j-1)}) \quad \text{and} \quad M_{k(i,j)} = M_{k(i,j-1)} + \sum_{k} \Delta M_{k(i,j)}$$  \hspace{1cm} (13)

We call the final results obtained using these equations as the second order static internal loads. Considering the stiffness obtained in final iteration of P-Delta method we compute the modes and frequencies of vibration of the structure and so accomplish the dynamic analysis, according to described by equations (7) and (8). We considered that the structure displaces around the equilibrium position given by the P-Delta Method.
3 STRUCTURE ANALYZED AND NUMERICAL RESULTS

The structure analyzed here is an RC telecommunication tower with 40 m long and diameter of 60 cm. The structure is cylindrical with cross-section in circular ring. Properties changes along the structure axis, because the thickness and steel area vary along the axis. The concrete used in the fabrication of the structure presents characteristic resistance (fck) at 28 days old equal to 45 MPa, what represents, according to Eq. (9) $E_{sec} = 41.4$ GPa. We consider the elasticity modulus of the structure $E = E_{sec}$. The concrete covering is 25 mm. The concrete design resistance is $f_{cd} = 45/1.3$ MPa. The steel conformed in structure presents $f_{y} = 500/1.5$ MPa (steel design stress) and $E_s = 210$ GPa. The structure is discretized into 40 elements of one meter long each one. The properties are shown in Table 1.

![Table 1. Structure properties](image)

In Table 1 we used the following notation: Node – the node number in the Finite Elements Method (FEM) Program; Height – level related to the ground level; Õext – external diameter of the cross-section; Thick. – thickness of the cross-section; M – nodal mass (lumped mass); A total – cross-section area; lc – moment of inertia of the circular ring; nb – is the number of longitudinal bars of the reinforced concrete section; Õb – diameter of longitudinal bars; As – is the total longitudinal steel area; Rb – is the radius of the circle that pass along the longitudinal bars axis; Is – is the total moment of inertia of the steel area; Is hom – is the homogenized total mo-
moment of inertia of the steel area; \( I_{\text{total}} \) - is the total homogenized moment of inertia of the reinforced concrete cross-section; \( I_s/ I_{\text{total}} \) - is the lower boundary value for \( w \) in each section.

According to NBR-6123-87 (ABNT, 1987), we consider the basic wind speed of \( V_0 = 35 \text{ m/s} \), the topographic factor is \( S_1 = 1 \), terrain roughness category IV, class B, what gives \( S_2 = (b; \rho; Fr) \) and the statistic factor is \( S_3 = 1, 1 \). As we stated before, the wind load on an area \( A \) is 
\[
F = g \cdot C_a \cdot A,
\]
where \( C_a \) is the aerodynamical coefficients. Several equipment are installed on the structure, they are: stairway with anti-falls cable, platform with antennas supports, night signer lights, protection against atmospheric discharges system and installed antennas. The values of \( A \) and \( C_a \) are: tower, \( 0 \leq z \leq 40 \text{ m} \), \( A = 0,6 \text{ m}^2/\text{m} \) and \( C_a = 0,6 \); stairways, \( 0 \leq z \leq 40 \text{ m} \), \( A = 0,05 \text{ m}^2/\text{m} \) and \( C_a = 2 \); cables, \( 0 \leq z \leq 40 \text{ m} \), \( A = 0,15 \text{ m}^2/\text{m} \) and \( C_a = 1,2 \); platform and antennas supports, \( z \leq 40 \text{ m} \), \( A = 1 \text{ m}^2 \) and \( C_a = 2 \); antennas, \( z = 40 \text{ m} \), \( A = 3 \text{ m}^2 \) and \( C_a = 1 \). Table 2 shows the nodal mass and area for the structure analyzed.

Based on the results obtained by Brasil and Silva (2004) in this section we adopt the following equation (Fig.2) for the effective stiffness parameters:
\[
w = -1,5x^3 + 3,3x^2 - 2,5x + 1,1 \text{ and } w_i \leq w \leq 1, \text{ para } i = 0,1,...,n
\]  
\[\text{(14)}\]

![Figure 2. Effective stiffness adopted](image)

Note that upper is equal 1,0 and lower values vary. Because of the safety coefficients adopted materials and design process, usually in tests structures present values of \( x = M_u/M_b \geq 1,0 \). For a 30 m structure tested by Brasil and Silva (2004), the maximum value assumed by \( x \) was 1,33 and for other similar 40 m structure the maximum was \( x = 1,53 \).

Considering the lumped mass given in Table 1, the total homogenized moment of inertia for the LDA model and the effective moment of inertia of the final iteration of P-Delta Method for the NDA, we compute the natural modes and frequencies of vibration (Fig. 3). Note that in non-linear model the frequencies are smaller than in LDA. The coefficient of amplification \( \xi \) presented values until 2.35 for LDA and 2.65 for NDA.
The values obtained for bending moment in both models analyzed are shown in Fig. 4. In this figure we can see the following bending moments obtained: $M_{\text{lsa}}$ (LSA), $M_{\text{lda}}$ (LDA), $M_{\text{nnda}}$ (NDA), $M_d$ (design bending moment) and $M_t$ (bending moment applied in tests). The LDA presented values of bending moment 1.3 times those given by the LSA, while the NDA presented values 1.5 times those from LSA. The design moments are 1.4 times those given by LSA and 1.1 times those given by LDA. Comparing the results we conclude that the design bending moment is satisfactory to the LDA, but is not satisfactory for NDA. Other important conclusion is about the excellent performance of the structure related to the safety coefficient near to the failure. The structure resisted a load around 1.53 times the design moment. As we stated before, this is due to the safety coefficients applied on material strength. Results from tests show that the structure resists satisfactorily the bending moments given by NDA.
Others important considerations here are related to the elasticity modulus of concrete. In this work we considered \( E = 41.4 \, GPa \), computed according to NBR-6118-78 (ABNT, 1978) Brazilian Code. This value is larger than values measured in tests, around 21 \( GPa \), and larger than the value given by the revision of that Code, the new NBR-6118-03 (ABNT, 2003), around 31.9 \( GPa \) for the adopted concrete. Tests showed that when we compute a certain function \( w_1(x) \) considering a given elasticity modulus of concrete \( E_1 \) and solve the problem again using another value \( E_2 \), the new value of \( w \) is \( w_2(x) = \frac{E_1 w_1}{E_2} \), in other words, the quantity \( E_1 w_1 = E_2 w_2 = E w \) is a constant for different values of \( E \) adopted.

\section*{4 CONCLUSIONS}

In this work we propose a Non-Linear Dynamic model based on experimental data and the discrete dynamic model given by NBR-6123-87 (ABNT, 1987). We adopted the effective stiffness concept to represent the physical non-linearity and used a P-Delta Method to compute the geometrical non-linearity. We considered a cubic equation to represent the effective stiffness. We accomplished the NDA considering the effective stiffness in function of the strength level in each iteration of the P-Delta Method. The effective stiffness obtained in final iteration of P-Delta Method was used to compute the natural frequencies and modes of vibration. We considered that the structure displaces around the equilibrium position given by P-Delta Method. Finally, we computed the sum of non-linear static and dynamic strength. We compared the values obtained from NDA with those from LSA and LDA. The LDA presented values of bending moment 1.3 times those given by the LSA, while the NDA presented values 1.5 times those from LSA. The design moments are 1.4 times those given by LSA and 1.1 times those given by LDA. We conclude that the design bending moment is satisfactory to the LDA, but is not satisfactory for NDA. Results from tests show that “in practice” the structure resists satisfactorily the bending moments given by NDA.

Suggestions for future works are:

• process this structure considering different equations for the effective stiffness;
• accomplish this NDA using the synthetic wind method (Franco, 1993).

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\section*{REFERENCES}